**Barron’s Let’s Review Regents – Algebra I**

# Chapter 10: Functions

## 10.1 Describing Functions

A *function* is like a machine that converts numbers into other numbers. Functions are often named with the letters f, g, and h. If function f converts the number 2 into the number 7, we write f(2) = 7. Functions can be described as an equation, a graph, or a list of ordered pairs.

**What is a Function?**

A function can be thought of as a machine with an “in” slot into which you can put numbers on a card and another “out” slot from which numbers on cards come out.

Representing Functions as a List of Ordered Pairs

One way to know more about the function is if a set is given like this:

The ordered pair (1, 4) in the list indicates that if you put 1 into the “in” slot of the function machine, the number 4 will come out of the “out” slot.

**Domain and Range of a Function Described by a Set of Ordered Pairs**

The function f is defined only for the values 1, 2, 3, 4 and 5. If any other number is put into the “in” slot of this function machine, it will be *undefined*. These seven numbers are said to be in the *domain* of function f. The domain is usually written as a set of values. The domain of function f is {1, 2, 3, 4, 5 }.

The only values that can come out of a function’s “out” slot is called the *range* of the function.

**Domain and Range in Real-World Applications**

In some real-world situations, certain numbers don’t make sense. For example, you can’t buy half of a car or each negative 5 pieces of pizza. When a function is related to a real-world situation, there may be limits on the domain and range.

**When a Set of Ordered Pairs Cannot Describe a Function**

If in a function f, f(2) = 7, this means that any time the number 2 is put into the function machine, the number 7 comes out. There will never be a time that 2 is put in and the number 8 comes out. If it did, we would want to get our function machine repaired.

When a function is described by a list of ordered pairs, no two ordered pairs will have the same first coordinate. If (2, 7) is in the set, there will not be a   
(2, 8) or a (2, 9) or anything else with a 2 as the first coordinate.

A function cannot have different output values for the same input value.

### Check Your Understanding of Section 10.1

1. Multiple-Choice
2. If a function f is defined as f = {(1,2), (2,3), (3,1), (4,4)}, what is f(2)?  
   **(3) 3**
3. If a function g is defined as g = {(1, 4), (3, 2), (4,3), (5, 1) }, what is g(2)?  
   **(4) Undefined**
4. If f(4) = 7, which could not be the definition of the function?  
   **(3) f={(1,3), (4,8), (5,6), (7,4)}**
5. If f = {(1,4), (2,8), (3,7), (4,1)} and g = {(1,5), (2,9), (3,6), (4,2)}, which of the following is true?  
   **(3) f(3) > g(3)**
6. Which of the following *cannot* be the definition of a function?  
   **(1) f = {(1,5), (2,7), (2,8), (4,9)}**
7. What is the domain of the function defined as f = {(1,4), (3,7), (4,8), (5,8)}?  
   **(3) {1, 3, 4, 5}**
8. What is the range of the function defined as   
   f = {(1,4), (3,7), (4,8), (5,8)}?  
   **(4) {4, 7, 8}**
9. If f = {(1,4), (2,3), (3,2), (4,1)} and g = {(1,3), (2,4), (3,1), (4,2)}, what is f(g(1))?  
   **(2) 2**
10. A function g takes as an input a number representing the number of gallons of gasoline purchased and outputs the price of that many gallons of gasoline. What is the domain of this function?  
    **(1) All numbers greater than or equal to zero**
11. A function t takes as input the day of the year in New York and outputs the average temperature for that day. What is a reasonable range for this function?  
    **(1) Numbers between -10 and 100**
12. Show how you arrived at your answers
13. Explain why this is or is not a definition of a function. f = {(3,5), (4,5), (5,5), (6,5)}  
      
    **A function is like a machine that converts numbers into other numbers.   
      
    When a function is described by a list of ordered pairs no two ordered pairs will have the same first coordinate.**  
    **The definition of f qualifies as a function.**
14. If f = {1,4), (2,1), (3,2), (4,3)}, calculate   
    (a) f(1) + f(2); (b) f(1+2); (c) f(f(1)).  
      
    **(a) f(1) + f(2) = 4 + 1 = 5  
    (b) f(1+2) = f(3) = 2  
    (c) f(f(1)) = f(4) = 3**
15. If f = {(1,3), (4, 9), (5,2), (6,8)} and f(a) = 8, what are all possible values for a?  
      
    **a = 6**
16. Is f = {(3,4), (3,5), (4,7), (5,1)} a function? Explain why or why not.  
      
    **When a function is described by a list of ordered pairs no two ordered pairs will have the same first coordinate.**  
    **This is not a function because there are two ordered pairs with the same first coordinate (3). f(3) cannot be both 4 and 5.**
17. William says the range of a function always has the same amount of numbers or more numbers than the domain. Mia says the range of a function can have either the same amount of numbers as the domain or fewer numbers than the domain. Alice says that the range of a function can have more numbers, fewer numbers, or the same amount of numbers as the domain. Which of the three students is correct and why?  
      
    **Mia is correct.**  
    **A function with fewer numbers in the range than the domain:  
    f = {(3,5), (4,5), (5,5), (6,5)}  
    domain: { 3, 4, 5, 6 }  
    range: { 5 }  
      
    A function with the same amount of numbers in the range as in the domain:  
    f = {(3,3), (4,4), (5,5), (6,6)}  
    domain: { 3, 4, 5, 6 }  
    range: { 3, 4, 5, 6 }  
      
    There cannot be a range that has more numbers than are in the domain because:  
      
    When a function is described by a list of ordered pairs no two ordered pairs will have the same first coordinate. Repeated x-values are not permitted for a function.**

## 10.2 Function Graphs

A list of ordered pairs is not always the most efficient way to describe the input and output values of a function. A graph of these values is especially useful when there are an infinite amount of numbers in the domain of the function. The function graph can be used to evaluate the function at different values and can also be used to determine the domain and range of the function.

Graphing a Function

If a function is represented as a set of ordered pairs, those ordered pairs can be graphed to form the graph of a function.

If function f is described as f = {(1,1), (2,3), (3,5), (4,7), (5,9)}, the graph will contain just five points corresponding to the five ordered pairs in the set. The graph of these five points looks like this.

A graph with blue dots and numbers

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Sometimes you are given the graph of a function and not the set of ordered pairs. From the graph, it is possible to answer questions about the function. For example, on the next page is the graph of a different function called f.

A grid with dots and numbers

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f(1) = 3  
f(2) = 1  
f(3) = 4  
f(4) = 2  
f(5) = 6  
f(6) = 5

**Determining If a Graph Represents a Function**

The set f = {(1,3), (1, 5), (2,6)} does not represent a function because two of the ordered pairs have the same first coordinate. f(1) can equal only one number, but from this set, it seems to be both 3 and 5. When these points from this non-function are graphed, the **two points with the same x-value lie on the same vertical line.**

If the graph of ordered pairs has at least two points that lie on the same vertical line, the graph fails the *vertical line test* and the set can not represent a function.

**Finding the Domain and Range from the Graph of a Function**

If the graph of a function is a bunch of points, the domain is the set of all the x-coordinates and the range is the set of all the y-coordinates of the points.

If the graph of the function is some kind of curve, the domain and range cannot be described by a list since there are an infinite number of values in each set.

There are an infinite number of points on a line segment.

For a line from (1,3) to (7,5), the domain can be described as and the range can be described as .

It is possible for the domain or the range to contain every real number. In this case, write “All real numbers.” A parabola might have a domain of all real numbers and a range where

### Check Your Understanding of Section 10.2

1. Multiple-Choice
2. Which is the graph of the function f = {(1,2), (2,5), (3,1), (4,1)}?  
   **(3)**
3. Below is the graph for which function?  
   **(1) f = {(1,1), (2,6), (3,5), (4,1), (5,2)}**
4. Below is the graph of y = f(x). What is the value of f(3)?  
   **(4) 4**
5. A portion of the graph of y = f(x) is show below. What is the value of f(100)?  
   **(1) 5**
6. Below is the graph of y = f(x). Which point could be used to determine the value of f(4)?  
   **(4) D**
7. Below is a graph of y = g(x). What is the approximate value of g(5)?  
   **(2) 6**
8. Below is the graph of y = f(x). What is the domain of f?  
   **(4) { 1, 2, 5, 6 }**
9. Below is the graph of y = f(x). What is the range of f?  
   **(1) { 1, 2, 3, 5 }**
10. Below is the graph of y = f(x). What is the domain and range of f?  
    **(2) Domain ,   
    range**
11. Which is the graph of a function?  
    **(3)**
12. *Show how you arrived at your answers*.
13. Alisha says that this is not the graph of a function because there are two points that have the same y-coordinate, like (3,9) and   
    (-3,9). Tyson says that it is the graph of a function. Who is right and why?  
      
    **Tyson is right because the graph passes the vertical line test, which requires there be only one point for any given vertical line. The vertical line test would fail if there are two points for any vertical line. Two   
    x-values may correspond to the same   
    y-value.**
14. What is the domain and range of the function whose graph is below?  
      
    **Domain:   
    Range:**
15. Using the two graphs below, determine the value of f(g(3)).  
      
    **g(3) = 5  
    f(5) = 2**
16. Below is the graph of y = f(x). What is the value of f(f(f(f(f(f(f(f(2))))))))?  
    f(2) = 2  
    f(f(2) = 2  
    …  
    **f(f(f(f(f(f(f(f(2)))))))) = 2**
17. In the graph of the function below, list all values that satisfy the equation f(a) = 4.  
      
    **a = -2, a = 2**

## 10.3 Defining a Function with an Equation

### Check Your Understanding of Section 10.3

1. Multiple-Choice
2. If , what is ?  
   **(4) 13**
3. If , what is ?  
   **(1) 11**
4. If , what is ?  
   **(3)**
5. If , what is ?  
   **(4)**
6. What is the average rate of change of the function over the interval   
   1 to 4?  
   Point 2: (4, 36)  
   Point 1: (1, 6)  
   **(3) 10**
7. If , which is the graph of   
   ?  
   Points: (0, -5), (2.5, 0)  
   **(4)**
8. If , for what value of a does ?  
   **(2) 3**
9. For what values of x is the graph of the function ?  
   **(3) All values**
10. What are the zeros of the function   
    ?  
    **(1) -2 and -8**
11. If , for what values of x is the function ?  
    **(1) For values between 1 and 5**
12. *Show how you arrived at your answers*.
13. If , what are (a) and   
    (b) ?  
    (a)   
    (b)
14. If and , what are  
    (a) and (b) ?  
    **(a)** **(b)**
15. and Manuel says that the expression can be written as *h(g(f(x)))*. Elizabeth says that this is not correct. Which student is right and why?  
      
    .  
      
    **Elizabeth is correct.  
    Manuel should have put parentheses around the expression before squaring.**
16. If and , what does the graph of g(x) look like?  
      
    A graph of a function

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17. If and , what is the value of ?

## 10.4 Function Transformations

The graph of the function looks very similar to the graphs of and . The only difference is that the graphs of the other four are translated – in other words, shifted left, right, up, or down. The graphs of the other four can be produced quickly if you know which direction to shift and what the original graph looks like.

**Vertical Translations**

The graph of will be the same as the graph of , only shifted up by *k* units.

The graph of will be the same as the graph of , only shifted down by *k* units.

**Horizontal Translations**

The graph of the function is the same as the graph of when translated 3 units to the left.

The graph of the function is the same as the graph of when translated 3 units to the right.

**Math Facts**

The graph of these four transformations will be congruent to the graph of the original function. The only difference is that it is shifted horizontally, vertically, or some combination of both. The four basic transformations are:

|  |  |  |
| --- | --- | --- |
| 1. |  | The graph shifts up by k units. |
| 2. |  | The graph shifts down by k units |
| 3. |  | The graph shifts left by k units. |
| 4. |  | The graph shifts right by k units. |

### Check Your Understanding of Section 10.4

1. Multiple-Choice
2. If below is the graph of , what is the graph of ?  
   **(4)**
3. If below is the graph of , which is the graph of ?  
   **(1)**
4. If below is the graph of , which is the graph of ?  
   **(2)**
5. If below is the graph of , which is the graph of ?  
   **(3)**
6. Below is the graph of on the left and of on the right. Which is equivalent to ?  
   **(4)**
7. Below is the graph of on the left and on the right. Which is equivalent to ?  
   **(1)**
8. Below is the graph of on the left and of   
    on the right. Which is equivalent to g(x)?  
   **(3)**
9. Below is the graph of on the left and of on the right. Which is equivalent to ?  
   **(2)**
10. If the graph of y=f(x) is a parabola with the vertex at (5,1), what is the vertex of the graph of the parabola ?  
    **(3) (7,1)**
11. If the graph of is below, which is the graph of ?  
    **(2)**
12. *Show how you arrived at your answers*.
13. Below is a chart of x-values and corresponding f(x)-values.   
    (a) If , what is the value of ?  
    (b) If , what is h(3)?

|  |  |
| --- | --- |
| x | y |
| 1 | 4 |
| 2 | 3 |
| 3 | 1 |
| 4 | 5 |
| 5 | 2 |

**(a)**   
**(b)**

1. Below is the graph of . Make a graph of

|  |  |  |  |
| --- | --- | --- | --- |
| x | f(x) | x: f(x-2) | y: f(x-2) |
| -5 | -2 | -3 | -2 |
| -1 | 4 | 1 | 4 |
| 1 | -2 | 3 | -2 |
| 5 | -2 | 7 | -2 |
| 7 | 0 | 9 | 0 |

A graph with a line

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A graph with a line going up

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1. The graph of has a domain of   
   .   
   (a) What is the domain of the graph of   
   ?  
   (b) What is the domain of the graph of  
   ?  
     
   **(a)   
   (b) Shifts left.**
2. The graph of is below. If   
   , and , make a graph of .  
     
    – shifts f(x) left 2 units  
    – shifts g(x) up 3 units

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | f(x) | x: g(x) | y: g(x) | x: h(x) | y: h(x) |
| 1 | 3 | -1 | 3 | -1 | 6 |
| 5 | 6 | 3 | 6 | 3 | 9 |
| 8 | 6 | 6 | 6 | 6 | 9 |

f(x)  
A graph with a line going up

AI-generated content may be incorrect.

g(x)  
A graph with a line going up

AI-generated content may be incorrect.  
  
h(x)  
A graph with a line going up

AI-generated content may be incorrect.

1. If , and   
   , how can g(x) be written as a transformation of ?